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EFFECT OF THE MAGNETIC FIELD UPON AMPLITUDE INCREASE OF ACOUSTIC WAVES IN A MEDIUM WITH DROPPING DENSITY

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EFFECT OF THE MAGNETIC FIELD UPON AMPLITUDE INCREASE OF ACOUSTIC WAVES IN A MEDIUM WITH DROPPING DENSITY

(O vliyanii magnitnogo polya na uvelicheniye amplitudy akusticheskikh voln v srede s padayushchey plotnost'yu)

(Presented on 23 December 1961 by Acad. Leontovich)

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by B. A. Tverskoy

1. It is well known, that with acoustic wave propagation in a medium with variable density, the velocity increases as the density drops. This result is easy to obtain in linear approximation. However, in the case of density dropping to zero, it remains in force only qualitatively, since the velocity approaches infinity and shock wave formation must be taken into account. The corresponding examination [1, 2] shows that even after shock wave formation acceleration continues through distances of the order of the length of the free path from the region with zero density. In the presence of a magnetic field and high conductance, acceleration must also extend to great amplitudes, for in that case, the equations would be similar to purely gas dynamic ones, while the characteristic length determining the limits of applicability of hydrodynamics is not the length of the free path, but the ion Larmor radius [3]. This conclusion is valid in case when magnetic pressure in unperturbed

regions drops to zero, just as the density does.

But if the magnetic pressure drops more slowly, it may be that the presence of the field would hinder the acceleration to high amplitudes, for the Alfvén velocity increases as the density drops, and, consequently — the rate of energy tapping also. Owing to that, the cause of acceleration, which is the energy transfer to an infinitely diminishing gas mass, should disappear.

2. In the presence of a magnetic field, the acceleration threshold may be estimated within the framework of linear approximation. Let us consider as an example the case of isothermal atmosphere in an homogenous gravitational field g. If we direct the axis z along the normal to the boundary plane, the balanced distribution (Boltzmann law) will be written in the form

$$\rho = \rho_0 e^{-z/z_0}, \quad p = p_0 e^{-z/z_0}, \quad z_0 = \frac{kT}{mg} \qquad (1)$$

Here ρ is the density, p — pressure, k — the Boltzmann constant, T — the absolute temperature and m — the mass of a molecule. We consider that the magnetic field H_0 is uniform and perpendicular to g. Assume that the conductance σ is sufficiently great, so that the condition of field freeze-in at oscillations is satisfied, and that dissipation, linked with the heat conductivity and the viscosity, is also immaterial.

Let us consider small oscillations of the atmosphere along the axis z with frequencies $\omega \ll eH_0/mc$ (e — being the ion charge, c — the speed of light). Under the proposed assumptions the problem in the entire space is described by magnetohydrodynamic equations (with a

precision to an immaterial variation of the adiabate index γ from 5/3 in dense to 2 in rarefied atmosphere layers.

For monochromatic oscillations, the equations will take the form:

$$i\omega v = -\frac{1}{\rho}\frac{dp'}{dz} - g\frac{\rho'}{\rho} - \frac{1}{4\pi\rho}H_0\frac{dH'}{dz}; \quad i\omega H' = -H_0\frac{dv}{dz};$$
$$i\omega\rho' + \frac{d}{dz}v\rho = 0; \quad i\omega\left(\frac{p'}{p} - \gamma\frac{\rho'}{\rho}\right) + v\frac{d}{dz}\ln p\rho^{-\gamma} = 0. \tag{2}$$

Here v, p', ρ', H' are respectively the velocity, pressure, density and magnetic field perturbations. The last equation expresses the adiabaticity of the motion.

Excluding all variables except v, and taking (1) into account, we shall have

$$(\varkappa^2 + e^{-\xi})\frac{d^2v}{d\xi^2} - e^{-\xi}\frac{dv}{d\xi} + e^{-\xi}\lambda^2 v = 0, \tag{3}$$

where

$$\varkappa^2 = \frac{H_0^2}{4\pi\gamma p_0}, \quad \xi = \frac{z}{z_0}, \quad \lambda^2 = \frac{\omega^2 z_0}{\gamma g}. \tag{4}$$

For $\xi = 0$, $v = v_0$, and the sources of oscillations must be absent at infinity.

In the absence of a magnetic field ($\varkappa^2 \equiv 0$) we have:

$$v = v_0 \exp\left[\tfrac{1}{2}\xi\left(1 - i\sqrt{4\lambda^2 - 1}\right)\right]. \tag{5}$$

This solution formally corresponds to cumulative acceleration ($v \to \infty$ for $\xi \to \infty$). The value of ρv^2 shall to the contrary remain limited (bounded). It is obvious though, that this result must be viewed as strictly qualitative, since for $v \geqslant \sqrt{\frac{kT}{m}\gamma}$ the applicability of the linear approximation is disrupted, and when $\rho \to 0$, the viscosity rises to infinity.

However for any small value of $\varkappa$, so long as it is not zero, the solution's behavior at infinity changes sharply.

Assuming

$$x = 1 + \frac{1}{\varkappa^2} e^{-\xi}, \tag{6}$$

we shall bring (3) to the hypergeometrical equation:

$$x(1-x)\,v'' + (1-2x)\,v' - \lambda^2 v = 0 \tag{7}$$

with parameters satisfying the correlations

$$\Gamma = 1, \quad \alpha + \beta = 1, \quad \alpha\beta = \lambda^2 \tag{8}$$

(to avoid confusion with adiabate indicator, we designated the third parameter by Γ in contrast to the generally-admitted γ).

In case $\lambda > 1/2$ is complex, and $\beta = \alpha^*$. Inasmuch as the solution is sought for in the region $x > 1$, it has the form [4]:

$$v_1 = \left(\frac{1}{x}\right)^{\alpha} F\left(\alpha, \alpha, 2\alpha, \frac{1}{x}\right); \quad v_2 = v_1^* \tag{9}$$

(F being the hypergeometrical function).

For $x \to 1$ ($\xi \to \infty$) both solutions diverge logarithmically. Let be $v_1 = \theta \ln(x-1) + \mathrm{Re}\, g$. The linear combination v_1 and v_2

$$v_3 = \mathrm{Im}\left\{\theta \left(\frac{1}{x}\right)^{\alpha} F\left(\alpha, \alpha, 2\alpha, \frac{1}{x}\right)\right\} \tag{10}$$

remains finite also at $x \to 1$. As a second solution (aside from v_3) we may take

$$v_4 = \mathrm{Re}\left\{\theta \left(\frac{1}{x}\right)^{\alpha} F\left(\alpha, \alpha, 2\alpha, \frac{1}{x}\right)\right\}. \tag{10'}$$

The obtained solution makes sense at $z \ll c/\omega$, since we neglected the displacement current. At greater distances the solution changes into the electromagnetic wave. As condition at infinity, one may impose the requirement that at $z_0 \ll z \ll c/\omega$ the solution pass into an electromagnetic wave travelling in the direction $z \to \infty$. the

The particle velocity in the wave is cE/H_0. But the electric field E in the travelling wave near $z = 0$ is $E_0 e^{i\omega(t-z/c)} \approx E_0 e^{i\omega t}(1 - i\omega z/c)$, i.e. with a precision to $\sim \omega z_0/c$, the velocity is constant. But if the source of oscillations is disposed at $z \to \infty$, the wave will be stationary and, inasmuch as by the strength of ideal atmosphere conductance, $E|_{z=0} = 0$, we shall have in the region $z_0 \ll z \ll c\omega$, $E \approx E_0 e^{i\omega t}\, \omega z/c$.

It is easy to show that for $z \gg z_0$ $v_3 = const$, $v_4 \sim z$. That is why in the absence of sources at infinity we must take the solution v_3. Thus:

$$v = v_0 \frac{\operatorname{Im}\{e^{-i\varphi}(1/x)^\alpha F(\alpha, \alpha, 2\alpha, 1/x)\}}{\operatorname{Im}\left\{e^{i\varphi}\left(\frac{\varkappa^2}{1+\varkappa^2}\right)^\alpha F\left(\alpha, \alpha, 2\alpha, \frac{\varkappa^2}{1+\varkappa^2}\right)\right\}}, \qquad (11)$$

where $\varphi = \arg \theta$.

At $\varkappa \ll 1$ and $x \ll 1$ (i.e. at $z \ll 2z_0 |\ln \varkappa|$)

$$v \approx v_0 e^{\xi/2} \frac{\sin(s\xi + 2s \ln \varkappa - \varphi)}{\sin(2s \ln \varkappa - \varphi)}, \qquad (12)$$

i.e. we have an acoustic type solution with a growing velocity (here $s = \frac{1}{2}\,\mathrm{Im}$). However, at $\xi \gtrsim 2|\ln \varkappa|$ acceleration ceases.

Let us note still another important distinction of magnetohydrodynamic oscillations from strictly acoustic ones: at

$$\omega \to \omega_n \approx \frac{g}{2}\sqrt{\frac{\gamma m}{kT}}\sqrt{1 + \left(\frac{n\pi + \varphi_n}{\ln \varkappa}\right)^2} \qquad (13)$$

resonance sets in. The computation of the phase multiplier φ_n is difficult. However, it may be shown that for small n, $\varphi_n \to 0$, and at great n, the role of φ_n is immaterial, since $|\varphi_n| \leqslant \pi$. The determination of resonance frequencies with the aid of the asymptotic formula (12) is valid at

$$n \lesssim \frac{1}{\varkappa |\ln \varkappa|} \gg 1.$$

The presence of resonances is related to magnetohydrodynamic wave reflection from regions with growing Alfvén velocity.

From physics of the problem (unlimited increase in the rate of magnetohydrodynamic energy tapping when density approaches zero), it is clear that these conclusions maintain their strength qualitatively, and this is also true in case of other laws of density drop, and in case of a field that is not perpendicular to g.

3. The results obtained may offer interest for certain problems of astrophysics. It is for instance well known that chromospheric flares on the Sun essentially take place near the neutral points of the magnetic field [5]. If chromospheric flares are linked with the egress of shock waves at the surface, which are excited in the outer convective zone of the Sun, the indicated law may me construed in the light of the above-said, as a consequence of cessation of wave acceleration in the medium with dropping density in the presence of a magnetic field. At the same time, the outmost acceleration will take place precisely near the neutral points, where the magnetic field is minimum.

In concusion, I avail myself of this opportunity to convey my deep gratitude to academician M. A. Leontovich, for a series of important comments.

***** E N D *****

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